

# **Design and Experimental Verification of Deployable/Inflatable Ultra-Lightweight Structures**

**Summary of Research**  
**Grant: NAG 1-01037 (6/1/2001-9/30/2004)**

Final Report to  
NASA Langley Research Center  
Hampton, VA23681

By  
Dr. P. Frank Pai  
Structural Mechanics and Controls Lab.  
Department of Mechanical and Aerospace Engineering  
University of Missouri-Columbia  
Columbia, MO 65211  
Phone: (573) 884-1474, Fax: (573) 884-5090, Email: [paip@missouri.edu](mailto:paip@missouri.edu)  
Web: <http://www.missouri.edu/~maepai>

1 November 2004

## **I. BACKGROUND**

Because launch cost of a space structural system is often proportional to the launch volume and mass and there is no significant gravity in space, NASA's space exploration programs and various science missions have stimulated extensive use of ultra-lightweight deployable/inflatable structures. These structures are named here as Highly Flexible Structures (HFSs) because they are designed to undergo large displacements, rotations, and/or buckling without plastic deformation under normal operation conditions. Except recent applications to space structural systems, HFSs have been used in many mechanical systems, civil structures, aerospace vehicles, home appliances, and medical devices to satisfy space limitations, provide special mechanisms, and/or reduce structural weight. The extensive use of HFSs in today's structural engineering reveals the need of a design and analysis software and a database system with design guidelines for practicing engineers to perform computer-aided design and rapid prototyping of HFSs. Also to prepare engineering students for future structural engineering requires a new and easy-to-understand method of presenting the complex mathematics of the modeling and analysis of HFSs. However, because of the high flexibility of HFSs, many unique challenging problems in the modeling, design and analysis of HFSs need to be studied.

The current state of research on HFSs needs advances in the following areas: (1) modeling of large rotations using appropriate strain measures, (2) modeling of cross-section warpings of structures, (3) how to account for both large rotations and cross-section warpings in 2D (two-dimensional) and 1D structural theories, (4) modeling of thickness thinning of membranes due to inflation pressure, pretension, and temperature change, (5) prediction of inflated shapes and wrinkles of inflatable structures, (6) development of efficient numerical methods for nonlinear static and dynamic analyses, and (7) filling the gap between geometrically exact elastic analysis and elastoplastic analysis.

The objectives of this research project were: (1) to study the modeling, design, and analysis of deployable/inflatable ultra-lightweight structures, (2) to perform numerical and experimental studies on the static and dynamic characteristics and deployability of HFSs, (3) to derive guidelines for designing HFSs, (4) to develop a MATLAB toolbox for the design, analysis, and dynamic animation of HFSs, and (5) to perform experiments and establish an adequate database of post-buckling characteristics of HFSs.

## **II. RESEARCH RESULTS**

### **1) Tasks Completed**

The following tasks have been completed during the three-year period:

- Developed a multiple shooting algorithm written in MATLAB for obtaining numerically exact large static or quasi-static deformations of highly flexible structures.
- Derived a geometrically exact membrane theory, a fully nonlinear forward analysis method using the multiple shooting algorithm, and a fully nonlinear inverse design method for the analysis and design of high precision large membrane structures.
- Derived a geometrically exact curved beam theory accounting for geometric nonlinearities and transverse shear deformations and a fully nonlinear forward analysis method using the multiple shooting algorithm for computing numerically exact large deformations of highly flexible beams.

- Enhanced the nonlinear finite element code GESA (Geometrically Exact Structural Analysis) by adding several new nonlinear elements.
- Performed fully nonlinear finite element analyses and dynamic tests on a tensioned rectangular Kapton membrane and an inflated Kapton tube using a scanning laser vibrometer and a 3D motion analysis system.
- Derived and performed geometrically exact finite element analysis of highly flexible plates and shells.
- Investigated the nonlinear modal couplings of highly flexible beams using a scanning laser vibrometer for measurement, and developed a new signal decomposition technique for examining modal couplings in nonlinear vibrations of HFSs.
- Investigated the nonlinear statics and dynamics of highly flexible structures using a newly purchased EAGLE-500 8-camera 3D motion analysis system. Especially we have studied the snapping dynamics of initially curved composite panels, nonlinear vibrations of Kapton membranes, large-amplitude vibrations of strings and beams, packaging and deployment dynamics of HFSs, and centrifugal stiffening and flutter of rotating beams.
- Designed and tested three different deployable unit cells that can be used to build large deployable/inflatable structures. They are 6-ring, 12-ring, and 32-ring unit cells.
- Built a test facility for performing large bending-torsion deformation tests on HFSs.

## **2) Significant Findings**

Significant findings are listed below:

- Geometrically exact structural theories can be derived by using Jaumann strains and stresses, exact coordinate transformations, and new concepts of local relative displacements and orthogonal virtual rotations.
- Numerically exact large static or quasi-static deformations of one-dimensional structures (e.g., cables, beams, and bars) and axisymmetric two-dimensional structures under axisymmetric loading can be obtained using the multiple shooting algorithm.
- All numerical results obtained from the GESA code agree well with the numerically exact solutions obtained from the multiple shooting algorithm.
- Nonlinear structural dynamics (such as 1:3, 1:2, and 1:2:3 internal resonances, energy transfer from a directly excited high-frequency mode to a low-frequency mode, and chaotic vibrations) of highly flexible structures can be characterized using a scanning laser vibrometer and the new signal decomposition method.
- Nonlinear statics and dynamics of highly flexible structures can be accurately measured and 3D instant distributions of internal stresses and strains can be estimated by using a 3D motion analysis system for measurement and a signal processing method for coordinate transformation and extraction of strain values.
- Packaging/deployment dynamics of deployable/inflatable space structures can be characterized using a 3D motion analysis system.

## **3) Future Work**

The following tasks are proposed for possible future work:

- To further enhance the GESA code by implementing advanced nonlinear cable, truss, beam, membrane, and shell elements. Large pre-buckling and post-buckling static deformations of more 1D and 2D HFSs need to be analyzed.
- To continue the large static deformation and buckling tests on different HFSs and to establish an adequate database of experimental post-buckling characteristics of basic components of HFSs.
- To study the influences of local plastic deformation and prestresses on the geometry, stiffness, and buckling strength of HFSs.
- To develop and test more packaging/deployment methods for basic components of deployable structures.

### **III. EDUCATION AND OUTREACH ACTIVITIES**

#### **1) Education**

Two Ph.D. and one M.S. students have graduated under the support of this grant. Moreover, two Ph.D. students are currently working on the modeling, design, analysis, and/or testing of HFSs.

#### **2) Outreach**

The following activities have been performed to disseminate the research results:

- Twelve papers have been presented at conferences.
- Four journal papers have been published, two papers are under review, and one paper is to be submitted soon.
- A seminar entitled "Highly Flexible Structures: Modeling, Computation and Experimentation" was given to the Structural Dynamics Branch of the NASA Langley Research Center on March 26, 2002.
- A seminar entitled "Fully Nonlinear Modeling, Analysis and Testing of Deployable/Inflatable Structures" was given to the Structural Dynamics Branch of the NASA Langley Research Center on April 11, 2003.
- A seminar entitled "Numerical and Experimental Techniques for Highly Flexible Structures" was given to the Structural Dynamics Branch and Aeroelasticity Branch of the NASA Langley Research Center on May 5, 2004.
- A web site has been established at <http://www.missouri.edu/~maepai> to show the activities and research results of this project.

### **IV. RESULTS AND IMPACT**

#### **1) Journal Papers**

1. Pai, P.F., Hu, J., and Belvin, W.K., "Packaging Analysis and Testing of Highly Flexible Triangular Frames," Int. J. Solids and Structures, to be submitted.
2. Pai, P.F., "Problems with Transverse Shear in Structural Analysis," Int. J. Computational Engineering Science, submitted.
3. Pai, P.F., "Total-Lagrangian Formulation and Finite Element Analysis of Highly Flexible Plates and Shells," Mathematics and Mechanics of Solids, submitted
4. Young, L.G., Ramanathan, S., Hu, J., and Pai, P.F., "Numerical and Experimental Dynamic Characteristics of Thin-Film Membranes," Int. J. Solids and Structures, in press (25 pages).

5. Pai, P.F. and Young, L.G., "Fully Nonlinear Modeling and Analysis of Precision Membranes," *Int. J. Computational Engineering Science* 4(1), 1-47, 2003.
6. Pai, P.F. and Lee, S.Y., "Nonlinear Structural Dynamics Characterization Using a Scanning Laser Vibrometer," *J. of Sound and Vibration* 264(3), 657-687, 2003.
7. Pai, P.F. and Lee, S.Y., "Three-Dimensional Postbuckling Analysis of Curved Beams," *Int. J. Computational Engineering Science* 3(3), 305-338, 2002.

## 2) Conference Papers

1. Pai, P.F., Hu, J., and Ramanathan, S., "Modeling, Analysis and Testing of a Deployable Triangular Frame," 46<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Austin, Texas, April 18-21, 2005.
2. Pai, P.F. and Hu, J., "Problems with Transverse Shear and Rotary Inertia in Structural Analysis," 46<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Austin, Texas, April 18-21, 2005.
3. Pai, P.F. and Hu, J., "Camera-Based Digital Real-Time Static and Dynamic Testing of Deployable/Inflatable Structures," 12<sup>th</sup> SPIE Annual International Symposium on Smart Structures and Materials, San Diego, California, March 6-10, 2005.
4. Hu, J. and Pai, P.F., "Design and Testing of Highly Flexible Deployable/Inflatable Structures," 21<sup>st</sup> Midwest Chinese American Science and Technology Conference, St. Louis, MO, August 21, 2004.
5. Ramanathan, S., Hu, J., and Pai, P.F., "Experimental Nonlinear Mechanics of Highly Flexible Structures Using a 3D Motion Analysis System," 7<sup>th</sup> International Conference on Motion and Vibration Control, St. Louis, Missouri, August 8-11, 2004.
6. Pai, P.F., Ramanathan, S., and Hu, J., "Static and Dynamic Characterization of Highly Flexible Structures Using an 8-Camera Motion Analysis System," 12<sup>th</sup> Chinese Vibration and Noise Conference, Taipei, Taiwan, June 4, 2004.
7. Young, L.G. and Pai, P.F., "Numerical and Experimental Dynamic Characteristics of Thin-Film Membranes," 45<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Palm Springs, California, April 19-22, 2004.
8. Pai, P.F. and Young, L.G., "Modeling, Analysis and Testing of Some Deployable/Inflatable Structures," 45<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Palm Springs, California, April 19-22, 2004.
9. Pai, P.F., "Mechanics and Dynamics Characterization of Highly Flexible Structures," 11<sup>th</sup> Chinese Sound and Vibration Conference, Keelung, Taiwan, July 12, 2003.
10. Pai, P.F. and Lee, S.Y., "Three-Dimensional Postbuckling Analysis of Highly Flexible Curved Beams," 44<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Norfolk, Virginia, April 7-10, 2003.
11. Pai, P.F., Lee, S.Y., and Young, L.G., "Experimental Nonlinear Structural Dynamics Using a Scanning Laser Vibrometer," 43<sup>rd</sup> AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Denver, Colorado, April 22-25, 2002.

12. Pai, P.F., Young, L.G., and Lee, S.Y., "Geometrically Exact Modeling and Design of High Precision Membranes," 43<sup>rd</sup> AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Denver, Colorado, April 22-25, 2002.

### 3) Theses

- Jiazhu Hu, Geometrically Exact Analysis and Dynamic Testing of Frames and Strings, Ph.D. student, 09/2003-present.
- Suhan Chong, Fully Nonlinear Design and Experimentation of Highly Flexible Structures under Twisting Loads, Ph.D. student, 09/2002-present.
- Suresh Ramanathan, Dynamics Characterization of Highly Flexible Beams Using a 3D Motion Analysis System, M.S. Thesis, July 2004.
- Leyland Young, Geometrically Exact Modeling, Analysis and Design of High Precision Membranes, Ph.D. Thesis, May 2003.
- Seung-Yoon Lee, Geometrically Exact Modeling and Nonlinear Mechanics of Highly Flexible Structures, Ph.D. Thesis, December 2002.

### 4) Computer Codes

- The finite element code GESA (Geometrically Exact Structural Analysis) is a total-Lagrangian displacement-based finite-element code for analyzing highly flexible structures. GESA is written in MATLAB and is based on the newly developed theories for structures undergoing large displacements, large rotations, and finite strains. The structural theories fully account for geometric nonlinearities due to large rotations, large in-plane strains of two-dimensional structures, large axial strains of one-dimensional structures, initial curvatures, and transverse shear deformations by using Jaumann stress and strain measures, an exact coordinate transformation, and a new concept of orthogonal virtual rotations. The Jaumann strains are derived using a new concept of local displacements without performing polar decomposition and they are proved to be a corotated geometric objective measure. Because all possible initial curvatures are included in the strain-displacement equations, governing equations of plates and shells are unified and the strain-displacement relations can be used for most one- and two-dimensional structures. For two-dimensional structures, only global translational DOFs and their derivatives are used in the strain-displacement relations and no independent global rotational DOFs are used. A corotated point reference frame is defined using the symmetry of Jaumann strains. Moreover, there is no need of transformation before updating strains, stresses, and displacements. Fully nonlinear cable, truss, beam, plate, and shell elements of different shapes and different numbers of nodes have been developed, and both isotropic and anisotropic materials are considered. Available experimental and numerical results show that GESA is accurate and efficient in computation.
- The computer code SHOOTING is for solving a set of nonlinear ordinary differential equations using direct numerical integration and the multiple shooting process to obtain numerically exact solutions.

### 5) Books

Many of the research results appear in the following two books:

- Pai, P. F., *Highly Flexible Structures: Modeling, Computation and Experimentation*, to appear in 2005.
- Nayfeh, A. H. and Pai, P. F., *Linear and Nonlinear Structural Mechanics*, John Wiley & Sons, New York, 2004 ([http://www.wiley.com/WileyCDA/WileyTitle/productCd-0471593567\\_descCd-authorInfo.html](http://www.wiley.com/WileyCDA/WileyTitle/productCd-0471593567_descCd-authorInfo.html), ISBN 0-471-59356-7, 746 pages).

#### **6) Patent Applications: none yet**

#### **7) Impacts**

This research project is expected to have significant and broad impacts on many aspects of structural engineering, especially the development of large deployable/inflatable space structures.

- This work solves several critical modeling issues and provide accurate modeling and analysis techniques for the analysis and design of HFSs.
- This work improves the accuracy and efficiency of analyzing nonlinear statics and dynamics of HFSs.
- This work reveals the post-buckling and nonlinear dynamic characteristics of HFSs.
- The derived design guidelines and database are useful for structural engineers to conceive preliminary designs of HFSs.
- The GESA code is an efficient tool for structural engineers to perform virtual experiments and have real-world insights into their designs, and hence rapid prototyping of HFSs can be done without trial and error or full-size testing.
- The GESA code can also be used in the design and analysis of MEMS devices and deployable/inflatable medical devices and implants.
- The GESA code has been used in a graduate/undergraduate finite element course and a graduate finite element course. Because of the concise and clear presentation of finite element procedures in the GESA code, students are able to easily understand and use this code in their projects and theses.